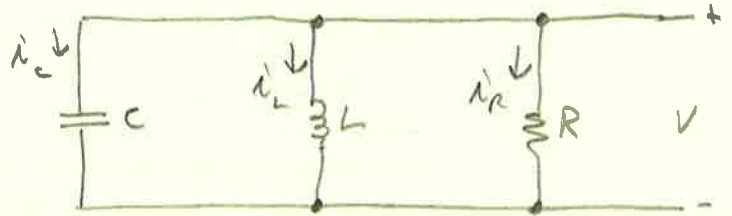


$$L = 10 \text{ mH}$$

for $t \geq 0$,

$$v(t) = 40e^{-1000t} - 90e^{-4000t} \text{ V}$$



a) Find ω_0 , α , C , and R

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1000$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -4000$$

adding, $-5000 = -2\alpha \Rightarrow \alpha = 2500 \text{ rad/s}$

$$-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1000$$

$$-2500 + \sqrt{2500^2 - \omega_0^2} = -1000 \Rightarrow \omega_0 = 2000 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = 25 \mu\text{F}$$

$$\alpha = \frac{1}{2RC} \Rightarrow R = 8 \Omega$$

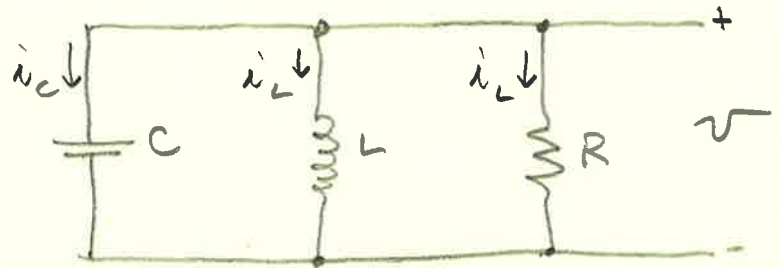
b) Find $i_R(t)$, $i_L(t)$, and $i_C(t)$ for $t \geq 0^+$

$$i_R(t) = \frac{v(t)}{R} = 5e^{-1000t} - 11.25e^{-4000t} \text{ A} \leftarrow$$

$$i_C(t) = C \frac{dv_C}{dt} = (25 \times 10^{-6}) \left[-10000e^{-1000t} + 90(4000)e^{-4000t} \right]$$

$$= -e^{-1000t} + 9e^{-4000t} \text{ A} \leftarrow$$

$$i_L(t) = -(i_R + i_C) = -4e^{-1000t} + 2.25e^{-4000t} \text{ A} \leftarrow$$



$$L = 10 \text{ mH}$$

$$v(t) = 40e^{-1000t} - 90e^{-4000t} \text{ V for } t \geq 0$$

a) FIND ω_0 , α , C , + R

system is overdamped

$$s_1 = -1000 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -4000 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

adding the 2 equations, $-5000 = -2\alpha$

$$\boxed{\alpha = 2500}$$

$$s_0 - 1000 = -2500 + \sqrt{2500^2 - \omega_0^2} \quad \text{or} \quad \boxed{\omega_0 = 2000 \text{ rad/s}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C = 25 \mu\text{F}}$$

$$\alpha = \frac{1}{2RC} \Rightarrow \boxed{R = 8 \Omega}$$

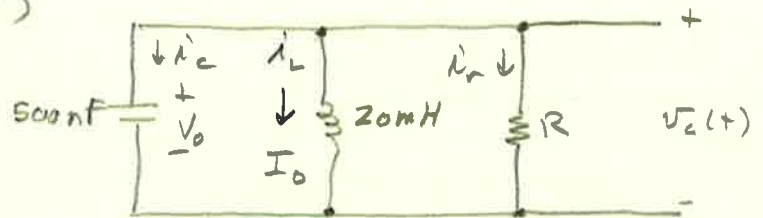
b) Find $i_R(t)$, $i_L(t)$, $i_C(t)$ for $t \geq 0$

$$i_R(t) = \frac{v_R(t)}{R} = \frac{v(t)}{R} = \boxed{5e^{-1000t} - 11.25e^{-4000t} \text{ A} = i_R(t)}$$

$$i_C(t) = C \frac{dv(t)}{dt} = \boxed{-e^{-1000t} + 9e^{-4000t} \text{ A} = i_C(t)}$$

$$i_L(t) = -(i_C(t) + i_R(t)) = \boxed{-4e^{-1000t} + 2.25e^{-4000t} \text{ A} = i_L(t)}$$

$V_c(0) = 40V$ (missing in text)
 $I_{0L} = 120mA$
 R adjusted for
 Critical Damping



a) Find R

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10,000$$

$$\alpha = 10,000 = \frac{1}{2RC} \Rightarrow \boxed{R = 100\Omega}$$

b) Find $V_c(t)$ for $t \geq 0$

$$V_c(t) = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$$

$$V_c(0) = 40 = A_2$$

$$V_c(t) = A_1 t e^{-10,000t} + 40 e^{-10,000t} \quad \checkmark$$

$$i_c(t) = C \frac{dV_c}{dt} = -.52A = C \left[A_1 t (-10,000) e^{-10,000t} + A_1 e^{-10,000t} + 40(-10,000) e^{-10,000t} \right]$$

$$-.52 = C [A_1 (-400,000)] \Rightarrow A_1 = -640,000$$

$$\boxed{V_c(t) = (-640,000t + 40) e^{-10,000t} \quad \checkmark}$$

c) Find $v(t)$ when $i_c(t) = 0$

$$i_c(t) = 0 \text{ when } \frac{dV_c(t)}{dt} = 0$$

$$i_c(t) = C \frac{dV_c}{dt} = C \left[-640,000t (-10,000) e^{-10,000t} + e^{-10,000t} (640,000) + 40(-10,000) e^{-10,000t} \right]$$

$$\frac{dV_c}{dt} = 0 \text{ when } -640,000t (-10,000) - 640,000 + 40(-10,000) = 0$$

$$t = 162.5 \mu s$$

$$\boxed{v(162.5 \mu s) = -12.60V}$$

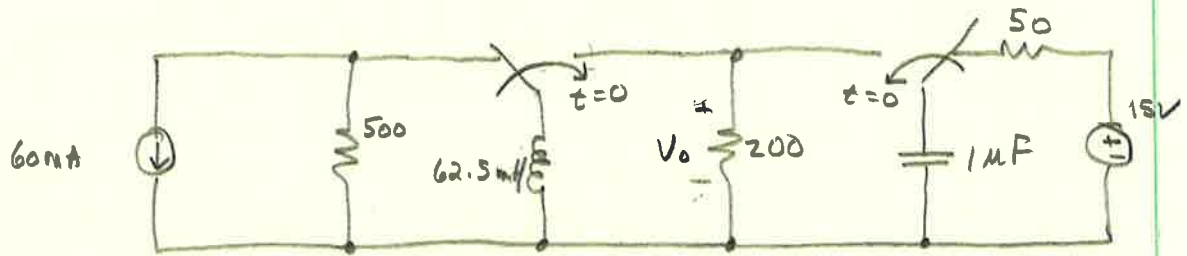
d) what % of initial energy is stored when $i_c(t) = 0$?

$$W_0 = \frac{1}{2} C V_0^2 + \frac{1}{2} L i_L^2 = 544 \mu J$$

$$W(162.5 \mu s) = \frac{1}{2} (C) (-12.6)^2 + \frac{1}{2} (102) \left(\frac{-12.6}{100} \right)^2 = 198.5 \mu J$$

$$\% \text{ remaining} = \frac{198.5}{544} \times 100\% = \boxed{36.5\%}$$

to find initial capacitor
 current, $V_c(0^+) = 40V$ which
 means $I_R = \frac{40}{100} = .4A \downarrow$
 $i_L = .12A \downarrow$ so $I_C = .52A \uparrow$



Find $V_o(t)$ for $t \geq 0$

1) Find I.C.'s before switches are flipped

$$i_L(0) = 60 \text{ mA}$$

$$V_o(0) = 15 \text{ V}$$

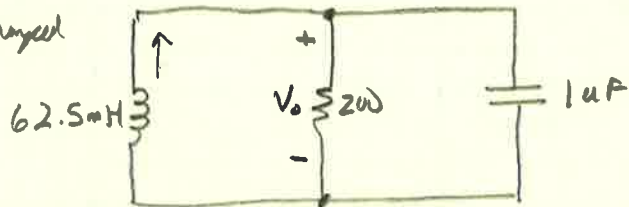
2) Find $V_o(t)$ for $t \geq 0$.

$$\sigma = \frac{1}{2RC} = 2500 \quad \left. \begin{array}{l} \text{underdamped} \\ \omega_0 = \sqrt{\frac{1}{LC}} = 4000 \end{array} \right\}$$

$$s_{1,2} = -2500 \pm j 3122.5 \text{ rad/s}$$

$$\omega_d = 3122.5$$

$$v_o = B_1 e^{-2500t} \cos 3122.5t + B_2 e^{-2500t} \sin 3122.5t$$



① $v_o(0^+) = B_1 = 15$

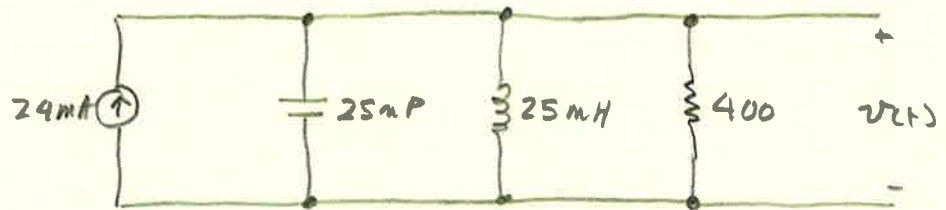
$$\frac{dv_o(t)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{C} \left(-\frac{V_o}{R} - I_0 \right)$$

$$-2500 B_1 + 3122.5 B_2 = -15,000$$

$$B_2 = 7.206$$

$$v_o(t) = 15 e^{-2500t} \cos 3122.5t + 7.206 e^{-2500t} \sin 3122.5t$$

Given:



$$i_L(0) = v_C(0) = 0$$

$$i_L(t) = (24 - 32e^{-20,000t} + 8e^{-80,000t}) \text{ mA} \quad t \geq 0$$

a) Find $v(t)$ for $t \geq 0$

$$v_L = L \frac{di_L}{dt} = v(t)$$

$$= (0.025) [640,000 e^{-20,000t} - 640,000 e^{-80,000t}] \times 10^{-3}$$

$$v(t) = 16 [e^{-20,000t} - e^{-80,000t}] \text{ V}$$

b) Find $i_R(t)$

$$i_R(t) = \frac{v_R(t)}{R} = \frac{v(t)}{R} = 40 [e^{-20,000t} - e^{-80,000t}] \text{ mA}$$

c) Find $i_C(t)$

$$i_C(t) = C \frac{dv_C(t)}{dt} = 25 \times 10^{-9} [16(-20,000) e^{-20,000t} + 16(80,000) e^{-80,000t}]$$

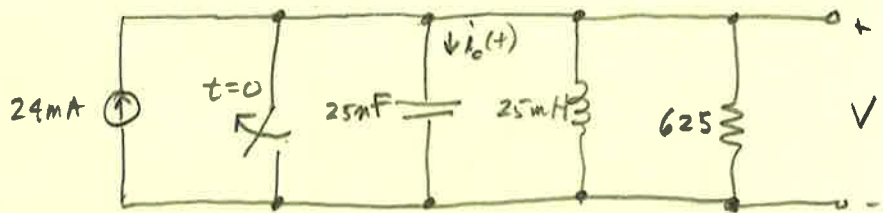
$$i_C(t) = -8 e^{-20,000t} + 32 e^{-80,000t} \text{ mA}$$

For $t \geq 0$, Find $v(t) + i_c(t)$

$$v_c(0) = i_c(0) = 0$$

$$i_c(\infty) = v(\infty) = 0$$

$$i_L(\infty) = 24 \text{ mA}$$



$$\alpha = \frac{1}{2RC} = \frac{1}{2(625)(25 \times 10^{-9})} = 32,000 \text{ rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25 \times 10^{-3}(25 \times 10^{-9})}} = 40,000 \text{ rad/sec}$$

$$\omega_0 > \alpha \text{ So roots are underdamped } \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 24,000 \text{ rad/s}$$

$$v_c(t) = v_{cf} + e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$= 0 + e^{-32,000t} (B_1 \cos 24,000t + B_2 \sin 24,000t)$$

$$v_c(0) = 0 = B_1$$

$$v_c(t) = e^{-32,000t} B_2 \sin 24,000t$$

$$i_c(0^+) = 24 \text{ mA} \text{ since } i_L \text{ can't change from } 0 \text{ @ } 0^- \text{ and } i_c \text{ changes instantaneously}$$

$$i_c(0^+) = \left. \frac{dv_c}{dt} \right|_{t=0^+} = C \left[e^{-32,000t} B_2 (24,000) \cos 24,000t + B_2 \sin \dots \right]$$

$$24 \text{ mA} = 25 \times 10^{-9} (24,000) B_2 \Rightarrow B_2 = 40$$

$$v_c(t) = 40 e^{-32,000t} \sin 24,000t \text{ V}$$

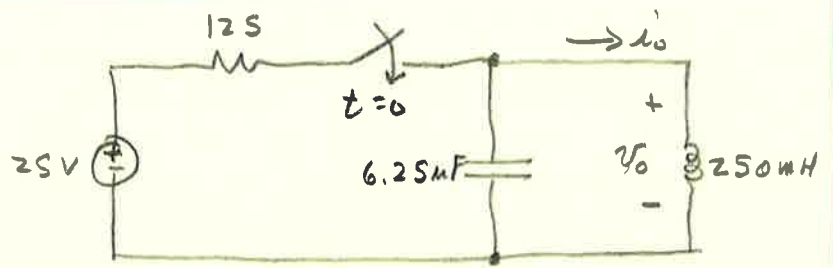
$$i_c(t) = \frac{dv_c(t)}{dt} = C \left[40(24,000) e^{-32,000t} \cos 24,000t + \sin 24,000t (40)(-32,000) e^{-32,000t} \right]$$

$$i_c(t) = 24 e^{-32,000t} \cos 24,000t - 32 e^{-32,000t} \sin 24,000t \text{ mA}$$

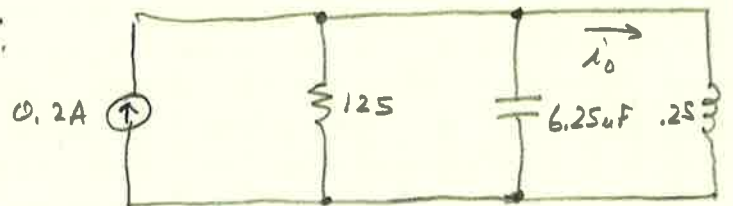
8.30

Niksson 11/11

$V_C(t) = i_L(t) = 0$
 FIND $i_o(t)$ for $t > 0$.



Do a source transformation:



$$\alpha = \frac{1}{2RC} = 640 \quad \omega_0 = \frac{1}{\sqrt{LC}} = 800$$

$\omega_0 > \alpha$ so response is underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 480$$

$$i_o = i_F + B_1 e^{-640t} \cos 480t + B_2 e^{-640t} \sin 480t$$

$$i_F = 0.2A$$

$$i_o(0) = 0 = 0.2 + B_1 \Rightarrow B_1 = -0.2$$

$$\begin{aligned} \frac{di_o}{dt}(0) &= B_1 [e^{-640t} (-\sin 480t)(480) + \cos 480t [-640 e^{-640t}]] \\ &\quad + B_2 [e^{-640t} \cos 480t (480) + \sin 480t (e^{-640t}) (-640)] \\ &= B_1 [-640] + B_2 [480] \end{aligned}$$

But $\frac{di_o}{dt}$ cannot change instantaneously so $= 0 @ t = 0^+$

$$\text{so } 0 = [-0.2(-640)] + B_2(480) \Rightarrow B_2 = -0.2667$$

$$i_o(t) = 0.2 - 0.2e^{-640t} \cos 480t - 0.2667e^{-640t} \sin 480t \text{ A } t > 0$$